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A MODIFIED DOUBLE MONTE CARLO TECHNIQUE TO APPROXIMATE RELIABILITY CONFIDENCE LIMITS OF SYSTEMS WITH COMPONENTS CHARACTERIZED BY THE WEIBULL DISTRIBUTION

THESIS

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RELIABILITY CONFIDENCE LIMITS OF SYSTEMS WITH

COMPONENTS CHARACTERIZED BY THE

WEIBULL DISTRIBUTION.

THESIS

Presented to the Faculty of the School of Engineering

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by

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Preface

A common practice in reliability engineering is to utilize available component sample data to derive a point estimate of the reliability of each component. This thesis is a continuation of previous work accomplished at the Air Force Institute of Technology on reliability estimation. Its purpose is to indicate the manner in which a modified Double Monte Carlo technique can be utilized to derive confidence interval estimations of system reliability based on sample component data.

I wish to express my sincere appreciation to Professor Jon R. Hobbs for his guidance and expertise toward completion of this study and to Professor Albert H. Moore for suggesting this topic and his encouragement and guidance throughout its development.

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Abstract

A digital computer technique is developed, using a modified Double Monte Carlo simulation, which determines lower confidence limits for system reliability based on component test data. This test data is assumed to have failure times which are distributed according to a known two-parameter Weibull probability distribution. The first step of the modified Double Monte Carlo technique is to randomly generate these failure times using the true shape and scale parameters of each component. The component distribution shape and scale parameters are then estimated by the method of maximum-likelihood from these component failure times. The second step of this technique is to reestimate these component shape and scale parameters using generated samples whose failures have the same distribution and parameters as the estimated ones and the same number of observations as the original test data. The method of maximum-likelihood is again used to estimate these component parameters. These twice estimated parameters are then substituted into the reliability equation to obtain the maximum-likelihood estimator for the component reliability. The estimated bias in this estimator is subtracted to yield an approximately unbiased estimator of component reliability. A given number of component reliabilities are obtained and used with the median rank values to construct an empirical distribution

for each component. The desired number of estimates are then sampled from these distributions to obtain a system reliability. Using this technique, where the distribution or joint distribution of the estimators is unknown, a Monte Carlo simulation is run for four hypothetical systems consisting of as many as five components. Since the true reliability is known, it can be determined if the desired confidence intervals contain the true system reliability. The result is a measure of the effectiveness of the modified Double Monte Carlo technique.

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I. Introduction

Problem Statement

In the Air Force today, there is a tendency towards increasing complexity of systems which correspondingly makes the achieving of high orders of reliability more difficult. Because of this increasing complexity and the concern over the ever increasing Defense budget, these reliabilities must be evaluated by using less testing and more efficient methods of reliability estimation. As a result, much effort has been made in establishing methods for predicting reliabilities of complex systems from their component test data.

Since the exact reliability of a component can not be measured directly, it must be estimated. These estimates can vary considerably in their accuracy; therefore, it is necessary that limits be attached to their probable range. When these limits are determined to a desired degree of confidence, reliability confidence limits result. It is meaningless to say that a system is 90% reliable, after testing its components, without some type of confidence limit. The purpose of this thesis is to determine these

reliability confidence limits based on test data about individual components that comprise the system.

Background

In 1960, Orkand (Ref 14) used a Monte Carlo method for determining lower confidence limits on system reliability. His study showed the limitations in using only point estimates of component reliability to determine a point estimate of the system reliability. Bernhoff (Ref 3) explored the problems of applying analytical approaches to establishing system reliability confidence limits. He concluded that system confidence limits can be obtained analytically if all components of the system have the same mathematical form for reliability. If the system reliability is a function of two or more dissimilar mathematical expressions, the only practical way of finding the approximate reliability distribution is to use a Monte Carlo technique (Ref 3:49).

Levy (Ref 8) developed a Monte Carlo technique where components were subjected to life tests and the mathematical model for component failures was assumed to be of the Exponential, Normal, Lognormal, Gamma, or Weibull distribution.

Moore (Ref 13) extended the Monte Carlo methods to include those cases where the joint distribution of the estimators is unknown. He was able to estimate their joint distribution by use of a technique he developed called Double Monte Carlo.

Moore and Levy (Ref 9) designed a digital computer technique for obtaining system reliability confidence limits where the system component failures exhibited different probability densities. It was assumed that the location parameter is zero or known, that the shape parameter is known and that the probability density function is given. With these assumptions, the exact distribution of the maximum likelihood estimator can be determined.

Lutton (Ref 10) assumed that the component life distributions were known. He generated reliability samples using the original Levy (Ref 8) technique and the asympotic distribution of parameter estimates of the Weibull, Gamma and Logistic density functions.

Lannon (Ref 7) established reliability confidence intervals for the Weibull distribution using a bivariate analysis with the scale and shape parameters unknown.

Most recently, Snead (Ref 18) studied reliability estimates using the Weibull, Gamma, and Logistic distribution with the property that the reliability estimators are asympotically normal. This study required large sample sizes to perform a univariate analysis using just the reliability parameter. Putz (Ref 15) further developed this univariate technique by simplifying the bivariate analysis of Lannon (Ref 7), by mapping the shape and scale parameters onto the reliability parameter and reducing the sample sizes of the component test data.

Rice (Ref 16) incorporated the asymptotic normality properties of the binomial distribution using a Monte Carlo technique for estimating lower confidence limits of system reliability. He also incorporated a technique developed by Gatliffe (Ref 4), which substituted equivalent failures in his component failure estimations, when the component exhibited no failures.

Darrel Thoman, Lee Bain, and Charles Antle (Ref 19) showed that, assuming the shape and scale parameters are unknown, location parameter is zero, in the two-parameter Weibull, that the distribution of the maximum likelihood estimator of reliability, depends only on the true system reliability and the sample size.

Reliability

The reliability of a system is defined as the probability that the system will be operating at some specified time (t) under specific conditions. If T is the time to failure or life length of a system or component, the reliability at time t or $R(t)$ is given by $R(t)=P(T>t)$, where P means "probability of". The system reliability is dependent on each component reliability and the system configuration. For example, if the components are connected serially, the failure of any one of them will cause the system to fail. If, on the other hand, the components are connected in parallel, the system will fail only if all components or a specified number of them fail.

The Weibull Distribution

The Weibull probability density function (pdf) was originally developed and used by Waloddi Weibull in 1939 in a study of the phenomenon of rupture in solids. Since that time, the Weibull pdf has been found to be useful for application in lifelength and reliability testing for many mechanical and electronic components. It is frequently assumed that electronic components are distributed according to the Exponential distribution. Zelen and Dannemiller (Ref 20:36) have pointed out that the Exponential distribution is generally not a robust approximation to the Weibull, especially if the shape parameter is greater than 1.

The Weibull density function is defined as

$$f(t;k,\theta,c) = \frac{k(t-c)^{k-1}}{\theta^k} e^{-\left(\frac{t-c}{\theta}\right)^k} \qquad k,\theta \geq 0;\ c \leq t \qquad (1)$$

$$= 0 \quad \text{elsewhere}$$

where θ is the scale parameter, k is the shape parameter, c is the location parameter and t is the time. The scale parameter affects the dispersion of the random variable t about its mean. The shape parameter determines whether the hazard function is increasing, decreasing, or time invariant; while the location parameter determines the origin (or guaranteed life). The Weibull distribution can be used to model the exponential density function, if the shape parameter is 1. It can also approximate the Normal distribution

and the Raleigh distribution when the shape parameter is scaled at 3.5 or 2.0 respectively. Figure 1 shows the flexibility of the Weibull to model Exponential, the Normal and the Raleigh distributions.

Assumptions

This thesis will concern itself with systems which have components described by the Weibull pdf. It is assumed that the components have previously been determined to be Weibull or that the Weibull pdf adequately models the components in the system.

It is assumed that the components of the system being analyzed fail independently. That is, failure of a given component within a system does not depend upon either failure or successful operation of any other component.

The location parameter is assumed to be known and will be set equal to zero for all cases.

It is assumed that all components have been life tested and that all unknown parameters of the life distribution have been estimated from the data.

Confidence Limits

It is known that statistical estimates are more likely to be close to the true value as the sample size increases. Thus, there is a close correlation between the accuracy of an estimate and the size of the sample from

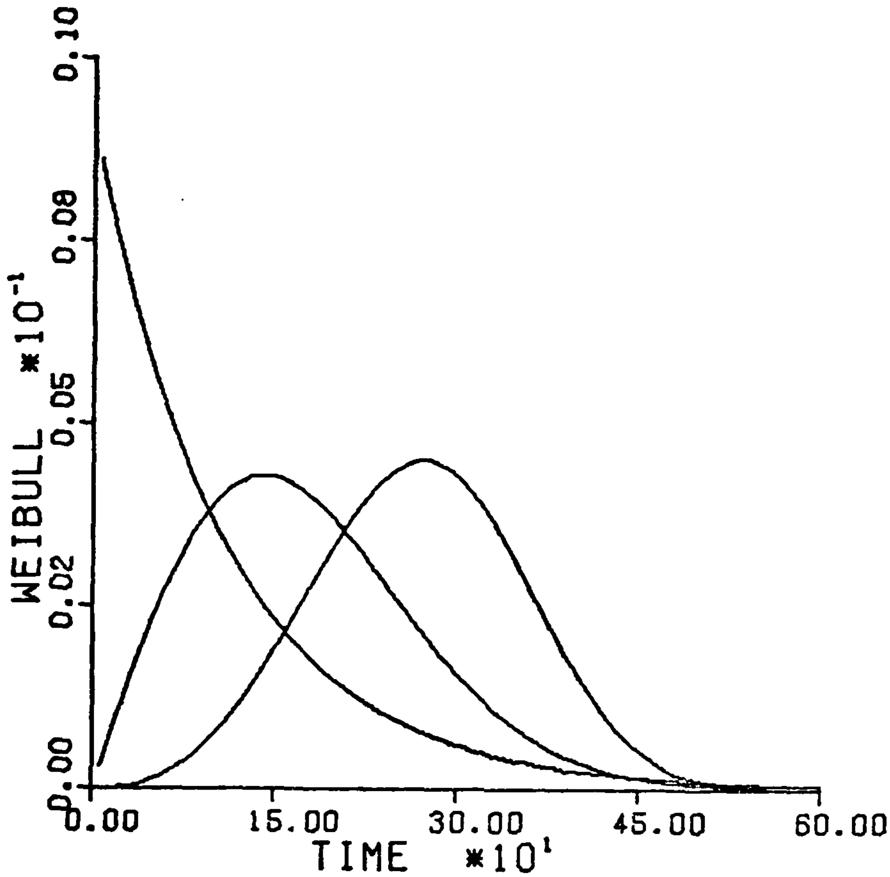

Fig 1. Weibull Distribution with Shape Parameters of 1, 2, and 3.5

which it was obtained. To this extent, in order to obtain a 100 percent confidence or certainty that a measured statistical parameter coincides with the true value, an infinitely large sample size or infinite interval is required.

When the estimate of a parameter is obtained from a reasonably sized sample, it may be logically assumed that

the true value of that parameter will be somewhere in the neighborhood of the estimate. Therefore, it is more meaningful to express statistical estimates in terms of a range or interval with an associated probability or confidence that the true value lies within such an interval, than to express them as point estimates. This is in fact what is accomplished when confidence limits are assigned to point estimates.

Confidence intervals around a point estimate have a lower confidence limit L and an upper confidence limit U . For example, if it is desired to calculate the confidence limits for a probability of 90 percent, this means that in approximately 90 out of 100 cases the true value will lie within the calculated limits, and approximately 10 cases will lie outside these limits. Distinguished from this confidence interval is a confidence level where it is assured that at some given level, say 10 percent, the true value lies within the calculated limits. If it is desired to increase the confidence level to 99 percent, so that in 99 percent of the cases the true value would lie within the confidence limits, the confidence interval around the point estimate would become much wider--or a much larger sample size must be used for the point estimate.

Objective

Compare the relative accuracy and utility of the modified Double Monte Carlo technique for finding system

reliability confidence limits with previously determined methods using the standard Double Monte Carlo method and the Univariate and Bivariate asymptotic distributions.

Approach

There must be a known mathematical relationship between the system reliability and component reliabilities (Ref 13: 459). Hence, the assumption that the components fail independently of one another. This allows use of standard formulas to calculate system reliabilities whether the components are connected in series or in parallel. Orkland describes procedures to apply if there is a dependence relation among system components (Ref 8:7-8).

A modified Double Monte Carlo technique will be used to determine reliability estimates and the associated confidence intervals of specific component networks. The unique feature of this method is that the distribution (joint distribution) of the estimator(s) for the parameter(s) can be unknown (Ref 13:461). The Monte Carlo method uses random sampling to investigate the solution of deterministic or stochastic problems. In essence, one inserts a set of random inputs from a specific probability distribution and solves the problem for each set of inputs to obtain a random sample of outcomes (Ref 13:459). The steps to this method are as follows:

1. Calculate the maximum likelihood estimators of the component shape and scale parameters from the component failure times. Do this for all components.

2. Samples are then generated whose failure have the same distribution and parameters as the estimated ones and the same number of observations as the original test data. These parameters are also estimated by maximum likelihood.

3. Substitute the sample parameter estimates into the component's reliability equation to derive a sample component reliability.

4. Repeat steps 1-3 to form an empirical distribution of component reliability estimates for each component.

5. Generate a point sample component reliability for each component from (4).

6. Calculate a point sample of system reliability from the point sample of component reliability.

7. Repeat steps 5-6 to obtain the desired number of samples of the system reliability.

8. Order the system reliability samples to obtain a sample cdf of system reliability from which the confidence limits are determined at a given confidence level.

The main advantage of the Monte Carlo method is its usefulness in solving very complicated problems. The disadvantage is in its slow convergence and resulting higher computer costs (Ref 13:459).

II. Theoretical Development

Estimators

An estimator is a rule which tells how to calculate the estimate of something, based upon information contained in the sample (Ref 2:161). One type of estimator is a point estimate where a single number, representing the estimate, may be associated with a point on a line. An example of a point estimate is the sample mean $\bar{y}$, where

$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$. This estimator of the population mean, μ , explains exactly how the actual numerical value of the estimate may be obtained *once the* sample values $y_1, y_2, \ldots, y_n$ are known.

A different problem with multiple parameter estimation may be stated as follows: Assume that $f(t, \theta_1, \theta_2, \ldots, \theta_k)$ is a density with k unknown parameters and let $t_1, t_2, \ldots, t_n$ be a random sample of size n . In the case of the Weibull pdf, there are three parameters k, θ, c , with $t_1, t_2, \ldots, t_n$ being sample failure times. The problem is to find a function of the observed samples which may be represented by $k(t_1, t_2, \ldots, t_n)$, $\theta(t_1, t_2, \ldots, t_n)$, $c(t_1, t_2, \ldots, t_n)$, such that the distribution of these functions will approximate, as closely as possible, the true values of the parameters. Each of these functions will be an estimator

of the true value and will be denoted as $\hat{k}$, $\hat{\theta}$, $\hat{c}$ respectively. (Note: In this thesis the location parameter c is always assumed to be zero.)

Method of Maximum Likelihood

A very general method of finding point estimates of parameters is the maximum likelihood estimator (MLE). This method selects those values of the parameters which maximize the probability or the joint density (the likelihood) of the observed sample (Ref 11:302). The likelihood function can be defined as follows: Let $y_1, y_2, \ldots, y_n$ be sample observations taken on corresponding random variables, $Y_1, Y_2, \ldots, Y_n$. Then if $Y_1, Y_2, \ldots, Y_n$ are discrete random variables, the likelihood of the sample, L, is defined to be the joint probability of $y_1, y_2, \ldots, y_n$. If $Y_1, Y_2, \ldots, Y_n$ are continuous random variables, the likelihood, L, is defined to be the joint density evaluated at $y_1, y_2, \ldots, y_n$ (Ref 11:303). For the Weibull density, the likelihood function can be represented as:

$$L = f(t_1, t_2, \ldots, t_n;\ k, \theta, c = 0) .$$

Since there is an assumption that the failure times are independent then $L = \prod_{i=1}^{n} f(t_i;\ k, \theta)$.

It is desired to determine $\hat{k}, \hat{\theta}$, the MLE for k and θ respectively, the procedure used in this thesis is as follows:

1. Take the partial derivatives of the natural logarithm, with respect to each parameter.

2. Set these derivatives equal to zero.

3. Solve simultaneous equations for the values of the parameters.

Since the Weibull pdf is

$$f(t;\ k,\theta,c=0) = \frac{k(t)^{k-1}}{\theta^{k}} e^{-\left(\frac{t_i}{\theta}\right)^{k}} \qquad k,\theta,t\geq 0 \tag{2}$$

then

$$L = k^{n}\theta^{-kn}\left(\prod_{i=1}^{n} t_i^{\,k-1}\right)\exp\left(-\theta^{-k}\sum_{i=1}^{n} t_i^{\,k}\right) \tag{3}$$

and

$$\ln (L) = n \ln k - nk \ln \theta + (k-1)\sum_{i=1}^{n} \ln t_i - \theta^{-k}\sum_{i=1}^{n} t_i^{\,k} \tag{4}$$

Taking the partial derivative of $\ln (L)$ with respect to θ and k ,

$$\frac{\partial \ln L}{\partial \theta} = -nk/\theta + k\theta^{-k-1}\sum_{i=1}^{n} t_i^{\,k} = 0 \tag{5}$$

$$\frac{\partial \ln L}{\partial k} = n/k - n \ln \theta + \sum_{i=1}^{n} \ln t_i - \theta^{-k}\sum_{i=1}^{n} t_i^{\,k} \ln t_i = 0 \tag{6}$$

Denoting the solutions by $\hat{k}$ and $\hat{\theta}$, Eqs (5) and (6) may be rewritten as:

$$\hat{\theta} = \left[\frac{\sum_{i=1}^{n} t_i^{\hat{k}}}{n} \right]^{1/\hat{k}} \tag{7}$$

$$\hat{k} = n/[1/\hat{\theta}^{\hat{k}} \sum_{i=1}^{n} t_i^{\hat{k}} \ln t_i - \sum_{i=1}^{n} \ln t_i] \tag{8}$$

Eqs (7) and (8) are two equations with two unknowns, $\hat{k}$ and $\hat{\theta}$. Simultaneous solution of these equations by an iterative procedure developed by Harter and Moore (Ref 5) yields the maximum likelihood estimators of k and θ.

Component Reliability

The reliability function, $R(t)$, may be expressed as:

$$R(t) = \int_t^{\infty} f(x)dx \tag{9}$$

where x is dummy variable of integration and $f(t)$ represents a failure density function. However, if the parameters of $f(t)$ are unknown and must be estimated from data samples, the reliability function itself must be expressed as an estimate; thus

$$\hat{R}(t) = g(t;\ \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k)$$

Since each component and their associated parameters are characterized by the Weibull distribution, the reliability function, $R(t)$, is determined using Eqs (2) and (9) as:

$$R(t) = \int_t^{\infty} \frac{k(x)^{k-1}}{\theta^k} \exp\left[-\left(\frac{x}{\theta}\right)^k\right] dx$$

which reduces to

$$R(t) = \exp(-(t/\theta)^k) \tag{10}$$

The MLE of reliability is found by substituting the MLEs for k and θ ($\hat{k}$ and $\hat{\theta}$) into Eq (10):

$$\hat{R}(t) = \exp(-(t/\hat{\theta})^{\hat{k}}) \tag{11}$$

Each of these estimated component reliabilities can then be used to estimate a system reliability.

System Reliability

Calculation of system reliability is achieved by using the laws of probability, given the reliability of each component. Since it is assumed that the components in each system or network being analyzed fail independently, the following equations can be used to obtain an overall system reliability $R_s(t)$:

[Note: Since the component reliabilities are estimates, the system reliability is also an estimate, $\hat{R}_s(t)$]

1. If the system is composed of two components connected in series, the reliability can be expressed by

$$\hat{R}_s(t) = \hat{R}_1(t)\cdot\hat{R}_2(t) \tag{12}$$

where $\hat{R}_1(t)$ and $\hat{R}_2(t)$ are component estimates.

2. If the system is composed of two components connected in parallel, the reliability can be expressed by

$$\hat{R}_s(t) = 1 - [1-\hat{R}_1(t)][1-\hat{R}_2(t)] \tag{13}$$

$$= 1 - \hat{Q}_1\hat{Q}_2 \qquad \text{where } \hat{Q}_1 = 1 - \hat{R}_i(t)$$

[Note: $Q(t)$ represents the probability that a component has failed]

More complex systems can be reduced to combinations of series and/or parallel configurations by use of Bayes' Theorem or the Boolean Disjunctive Theorem (Ref 7:5-6).

Bias of Reliability Estimate

An important criteria of an estimator is its bias. It is usually desired that the bias be as small as possible--approaching zero. The bias, B , is defined as the expected value of the estimator minus the true value of the parameter being estimated (Ref 11:266). That is

$$B = E[\hat{R}(t)] - R(t)$$

In a Monte Carlo simulation conducted by Thomas, Bain, and Antle (Ref 19:365), 10,000 estimates of $R(t)$ using sample

sizes of 8 to 100 and true reliabilities of 0.5 to 0.98 were estimated. The $E[\hat{R}(t)]$ was obtained by averaging the 10,000 estimates. Subtracting out the true reliability yielded the bias. Antoon (Ref 1:44) derived 2,000 estimates of $R(t)$ and calculated the bias for the same range of sample sizes and true reliabilities as Thomas, Bain, and Antle. Antoon's results compared favorably and are shown in Tables I and II.

<u>Median</u> <u>Rank</u> <u>Values</u>

Given an ordered random sample $y_1, y_2, \ldots, y_n$ from a population having a Weibull cumulative distribution function $F(y)$, where y is continuous, estimators can be determined for $F(y_1), F(y_2), \ldots, F(y_n)$. The distribution for these estimators, when used in life testing, is commonly termed the rank distribution. This rank distribution is derived in the reference and is denoted as follows (Ref 17: 297-298):

$$\frac{n!}{(j-1)!\,(n-j)!}\, p_j^{\,j-1} (1-p_j)^{n-j} dp_j \tag{14}$$

where $0 \le p_j \le 1$ and $p_j = F(y_j)$ is the fraction of the population failing prior to the jth ordered observation in a sample size n, which by differentiating yields

$$dF(x_j) = f(x_j)dx = dp_j$$

TABLE I

Bias of $\hat{R}(t)$ (Ref 1)

	Sample Size							
R(t)	8	9	10	11	12	13	14	15
.500	.00453	.00480	.00494	.00561	.00577	.00492	.00397	.00405
.550	.01067	.00952	.00944	.01061	.00976	.00767	.00688	.00610
.600	.00842	.00740	.00749	.00763	.00692	.00705	.00539	.00588
.650	.01508	.01267	.01232	.01162	.01088	.00899	.00836	.00806
.700	.01006	.01115	.01034	.00893	.00581	.00645	.00585	.00585
.750	.01064	.00926	.00881	.00816	.00821	.00781	.00728	.00635
.800	.01249	.01245	.01156	.01129	.01092	.01014	.00942	.00941
.850	.00670	.00632	.00530	.00556	.00493	.00597	.00583	.00483
.900	.00388	.00311	.00329	.00326	.00257	.00260	.00149	.00103
.925	.00233	.00384	.00304	.00268	.00200	.00187	.00132	.00208
.950	.00073	.00091	.00062	.00030	.00039	.00025	.00007	-.00026
.980	-.00246	-.00208	-.00232	-.00209	-.00192	-.00158	-.00139	-.00138

TABLE II

Bias of $\hat{R}(t)$ (Ref 1)

R(t)	Sample Size 20	25	30	40	50	75	100
.500	.00317	.00197	.00097	.00100	.00109	.00071	.00017
.550	.00519	.00388	.00307	.00367	.00310	.00275	.00203
.600	.00345	.00206	.00283	.00292	.00251	.00131	.00021
.650	.00632	.00499	.00369	.00292	.00251	.00131	.00021
.700	.00506	.00432	.00373	.00308	.00204	.00101	.00107
.750	.00459	.00386	.00432	.00343	.00213	.00184	.00054
.800	.00728	.00600	.00538	.00403	.00333	.00148	.00036
.850	.00453	.00262	.00223	.00222	.00114	.00158	.00149
.900	.00084	.00095	.00125	.00165	.00116	.00105	.00079
.925	.00103	.00089	.00004	.00044	-.00004	-.00035	-.00012
.950	.00075	.00070	.00018	.00010	.00015	.00004	.00028
.980	-.00118	-.00088	-.00075	-.00047	-.00029	-.00036	-.00027

The median value, commonly called the median rank, can be found by considering the integral:

$$P = \int_{0}^{\tilde{x}} g(p_j)\,dp_j$$

where $g(p_j)$ is defined in Eq (14). The value of $\tilde{x}$ for which $P=0.5$ would be the desired median value. An approximation to this median rank value that was used in this thesis is given by (Ref 17:300):

$$\tilde{x} = (j-0.3)/(n+0.4) \tag{15}$$

III. Procedure

This thesis presents a modified Double Monte Carlo method for obtaining confidence intervals for the reliability of various component networks for a specific mission time. This method will include the bias values and median rank derivation given in Chapter II.

Double Monte Carlo Method

The Double Monte Carlo Method for obtaining confidence limits for system reliability is used, as a technique, in cases where the distribution (joint distribution) of the estimator(s) for the failure model parameter(s) is unknown. It is assumed that (Ref 13): (1) the mathematical model for the underlying failure distribution is known; (2) a mathematical relationship relating system reliability and component reliability exists; and (3) the components of the system have been subjected to life tests and that the parameters of the failure model have been estimated.

Using the life test failure times for each component, and the estimated parameters of the failure model, samples are generated whose failures have the same parameters and distribution, Weibull in this case, as the known or estimated ones. It is important that these new samples have the same number of observations as the original test data. The estimates of the new parameters must be obtained via the

same method, maximum likelihood, and with the same sample size used on the original sample. These simulated values of the parameters along with a specified mission time are substituted in the life distribution to obtain a simulated reliability, denoted $\hat{\hat{R}}_i$, for each component.

In order to limit the amount of computer processing time, a given number of component reliabilities, $\hat{\hat{R}}_i$, will be estimated with the Double Monte Carlo method, for each component used in a given system. This given number of $\hat{\hat{R}}_i$ will be estimated for each Monte Carlo iteration. An empirical distribution will be formed for each component using the derived component reliabilities, $\hat{\hat{R}}_i$ ordered from smallest to largest, as the abscissae with an associated ordinate axis of ordered median ranks. Recall that these medium ranks are calculate using Eq (15) which is:

$$\tilde{x} = \frac{j - 0.3}{n + 0.4} \tag{15}$$

It should be noted that the first and last order statistic for each component reliability, associated with the median ranks 0 and 1 respectively, are approximated using linear extrapolation off of the two sequential order statistics nearest the first or last $\hat{\hat{R}}_i$. As an example of an empirical distribution see Figure 2. (Note: Each component reliability is determined with ten sample failures.) These distributions can then be used to obtain a large number of estimated component reliability.

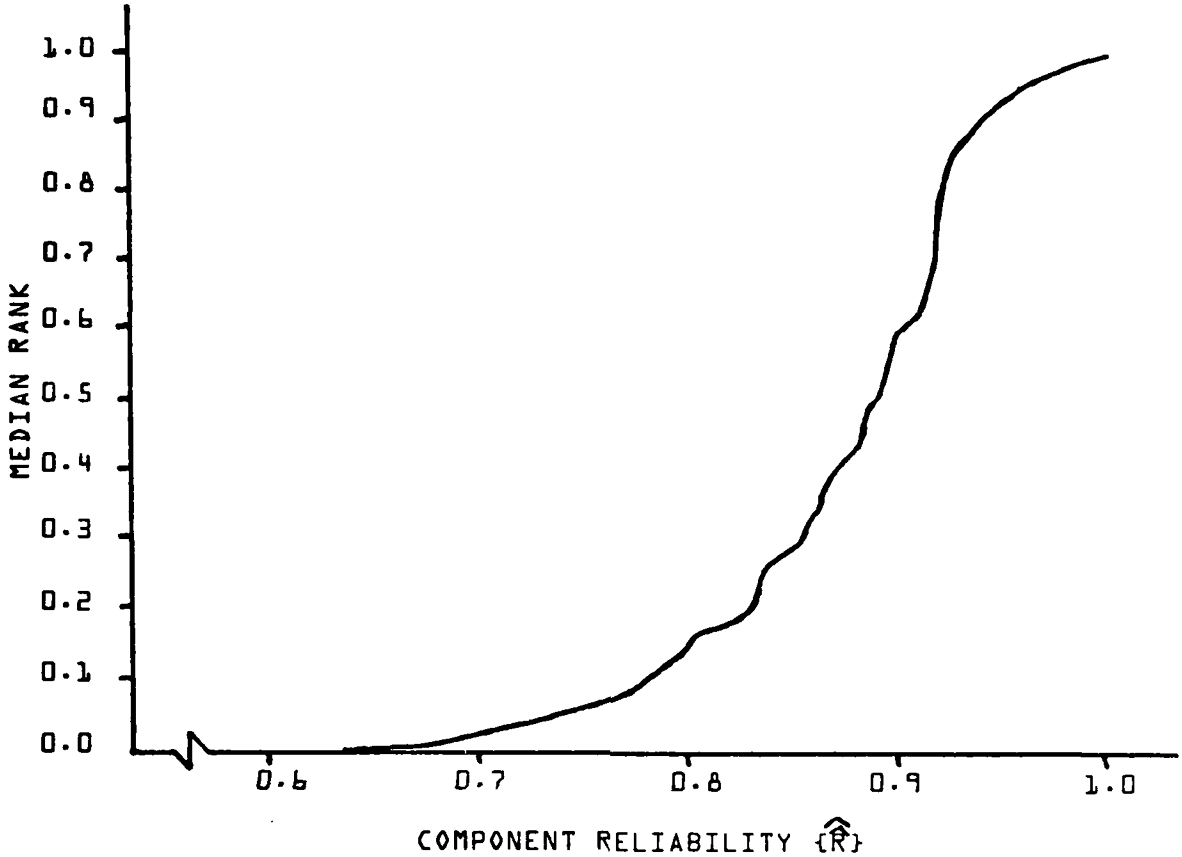

Fig 2. Sample Empirical Distribution with Shape = 2.0 and Scale = 250

Using the techniques specified in Chapter II, the true reliabilities can be calculated for each component and for each system. These true reliabilities will provide an absolute measure against which the modified Double Monte Carlo method can be gauged.

Calculating/Verifying System Confidence Limits

Given that sample values of system reliability have been generated, they are then ordered to yield a sample

cumulative distribution. Using this distribution, the confidence interval and limits can be approximated at any level of confidence. For example, in an ordered sample of 1,000 reliability points, the one hundredth value represents the lower limit of a one sided confidence interval at the 90% confidence level.

The steps for finding confidence intervals about the true system reliability, using the modified Double Monte Carlo method and verifying the accuracy of the confidence levels are as follows:

1. Using the true shape, scale and location parameters, generate a simulated sample of test data (component failure times) from the Weibull distribution.

2. Based on component test data, calculate the maximum likelihood estimators of the shape and scale parameters. The location parameter is assumed to be zero.

3. Samples are generated whose failures have the same distribution and parameters as the estimated parameters in (2) and the same number of observations as the original test data.

4. The parameters are again estimated from the simulated sample by the same method as used on the original sample (i.e., maximum likelihood estimates).

5. Substitute these simulated values of the parameters into the reliability function to obtain a maximum likelihood estimator of the component reliability.

6. Subtract the bias from the maximum likelihood estimator of the reliability, using Table I, to obtain an unbiased estimator of the component reliability.

7. Repeat steps 3-6 to obtain the desired number of component reliabilities for the empirical distribution.

8. Sample from this empirical distribution a given number of uniform random deviates to obtain the desired number of component reliability estimates.

9. Repeat steps 1-8 for each component.

10. Calculate point samples of system reliabilities from the point samples of component reliabilities.

11. Order the point samples of system reliabilities and determine the 99, 95, 90, 80, 70, 60, 50 percent one-sided confidence intervals. Note if each of these intervals contains the true system reliability.

12. Repeat step 1-11 until the desired Monte Carlo size is reached.

13. To measure the accuracy of the confidence limits, determine the percentage of the runs in which each of the confidence intervals covered the true system reliability.

Elaboration on each individual step of the technique is in Appendix A. A flow diagram and computer program which executes the above modified Double Monte Carlo method are shown in Appendices B and C respectively.

Components

Five components were considered in testing the modified Double Monte Carlo method. The true reliability of each component, R_i , is found by substituting into the Weibull reliability formula, Eq (11). Mission time, t , is arbitrarily set at 100 hours. The following components were used (Ref 15:22-23):

Component 1

Failure Distribution	Weibull
Parameter Values	$k = 2 \quad \theta = 250 \quad c = 0$
True Reliability	$R_1 = \exp[-(100/250)^2] = .85214$

Component 2

Failure Distribution	Weibull
Parameter Values	$k = 3 \quad \theta = 210 \quad c = 0$
True Reliability	$R_2 = \exp[-(100/210)^3] = .89765$

Component 3

Failure Distribution	Weibull
Parameter Values	$k = 2 \quad \theta = 300 \quad c = 0$
True Reliability	$R_3 = \exp[-(100/300)^2] = .89484$

Component 4

Failure Distribution	Weibull
Parameter Values	$k = 3.5 \quad \theta = 150 \quad c = 0$
True Reliability	$R_4 = \exp[-(100/150)^{3.5}] = .78512$

Component 5

Failure Distribution	Weibull
Parameter Values	$k = 2.5 \quad \theta = 250 \quad c = 0$
True Reliability	$R_5 = \exp[-(100/250)^{2.5}] = .90376$

Systems to be Analyzed

These five components were combined to form various kinds of systems. Four different systems were used and are shown in Figure 3.

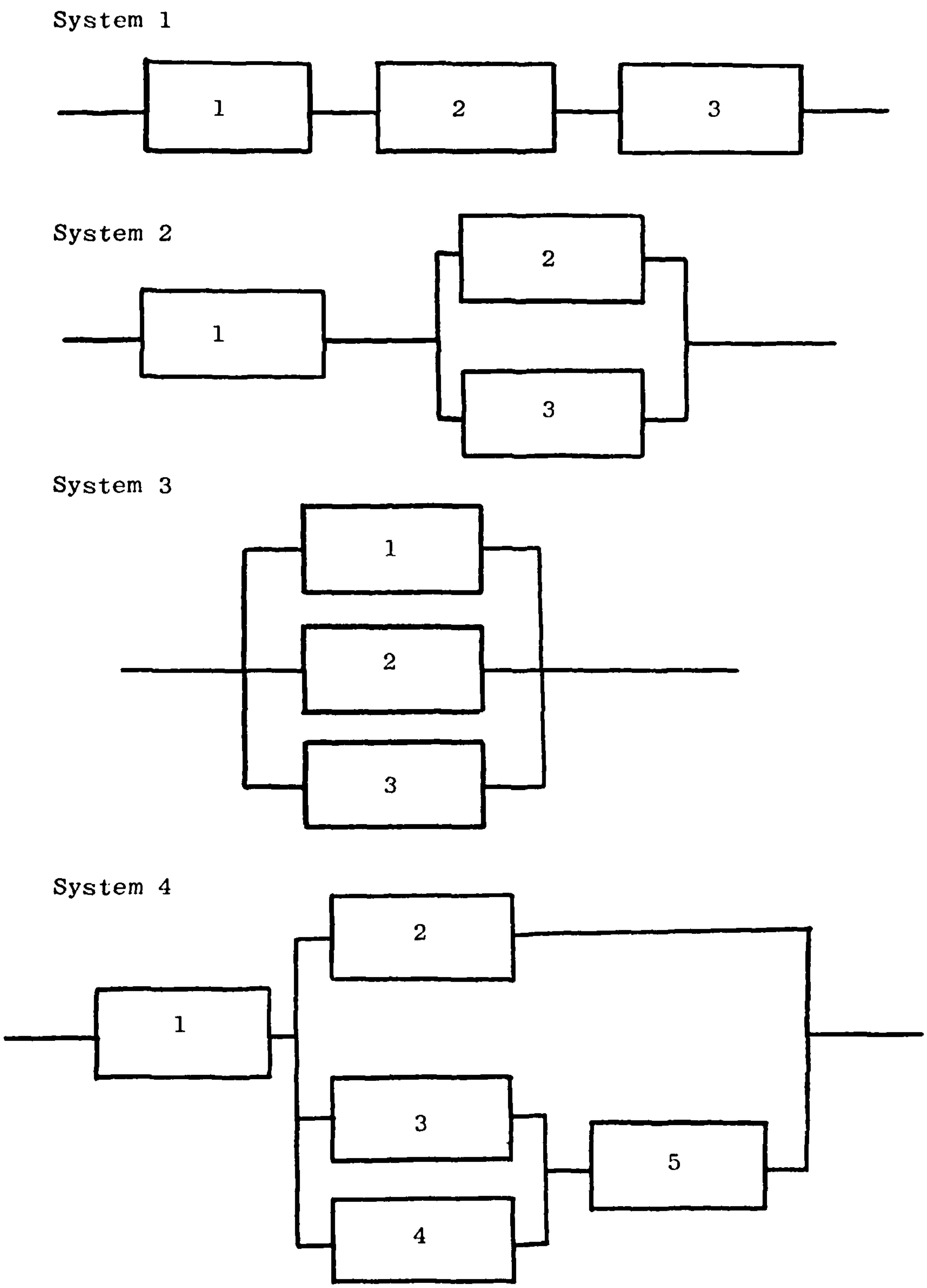

Fig 3. Systems 1, 2, 3, and 4 (Ref 15:24)

System 1 consists of three components in series. The true reliability of System 1, R_{s_1}, is

$$R_{s_1} = R_1R_2R_3 \tag{13}$$

$$= (.85214)(.89765)(.89484)$$

$$= \underline{.68448}$$

System 2 consists of one component connected in series with two in parallel. Let Q_i be the probability of failure for component i, then $Q_i = 1 - R_i$. Therefore,

$$R_{s_2} = R_1(1-Q_2Q_3) \tag{14}$$

$$= (.85214)[1-(.10235)(.10516)]$$

$$= \underline{.84297}$$

System 3 consists of three components connected in parallel. Hence,

$$R_{s_3} = 1 - Q_1Q_2Q_3 \tag{15}$$

$$= 1 - (.14786)(.10235)(.10516)$$

$$= \underline{.99841}$$

System 4 is a larger complex network of the previous systems, hence

$$R_{s_4} = R_1[1-Q_2[1-R_5(1-Q_3Q_4)]] \tag{16}$$

$$= .85214[1-.10235[1-.90376(1-(.10516)(.21488))]]$$

$$= \underline{.84197}$$

IV. Results

It is known that the error associated with a Monte Carlo calculation is proportional to $1/\sqrt{N}$ where N is the number simulations (Ref 17:259). In this case, the error is statistical, that is, the probable error is proportional to $1/\sqrt{N}$ or the probability is high that the approximate solution does not deviate from the true solution by more than a certain amount (Ref 5:22). The amount of statistical error in a calculation should decrease with the use of a high speed digital computer, but the computer can add random errors due to arithmetic roundoff.

Tables III through VI contain the results of the modified Double Monte Carlo simulation for Systems 1 through 4. The simulation was run for each system with component test data--sample size 10, 15, 20 and 30. In all cases run, the number of system reliability estimates used to partition the interval [0,1] was 1000. The empirical distribution for each component reliability was 75 points. The numbers specified at each table entry are the percentages of the Monte Carlo runs in which the true system reliability was contained within the simulated confidence interval. For example, the .9781 in Table III row 1 sample size 10, implies that in approximately 97 to 98 percent of

TABLE III

System 1 Confidence Interval Coverage of the True System Reliability

Confidence Interval (Percent)	Sample Size 10	15	20	30
99	.9781	.9831	.9910	.9922
95	.9203	.9372	.9371	.9429
90	.8771	.9699	.8702	.8797
80	.7454	.7667	.7689	.7921
70	.6092	.6317	.6209	.6441
60	.4666	.5010	.5011	.5171
50	.3891	.4237	.3788	.3626

TABLE IV

System 2 Confidence Interval Coverage of the True System Reliability

Confidence Interval (Percent)	Sample Size 10	15	20	30
99	.9600	.9592	.9709	.9777
95	.8781	.9000	.9213	.9346
90	.8003	.8419	.8671	.8779
80	.6811	.7003	.7118	.7135
70	.5602	.5617	.6177	.6389
60	.4765	.4883	.4890	.5049
50	.3617	.3812	.3900	.4371

TABLE V

System 3 Confidence Interval Coverage of the True System Reliability

Confidence Interval (Percent)	Sample Size 10	15	20	30
99	.8598	.9017	.9221	.9486
95	.7001	.7786	.7983	.8286
90	.5410	.6443	.6647	.7057
80	.3920	.4892	.4701	.5577
70	.2871	.3337	.3686	.4171
60	.1999	.2209	.2300	.3057
50	.1321	.1437	.1611	.2251

TABLE VI

System 4 Confidence Interval Coverage of the True System Reliability

Confidence Interval (Percent)	Sample Size 10	15	20	30
99	.9553	.9590	.9692	.9735
95	.8807	.9011	.9273	.9440
90	.8176	.8348	.8600	.8900
80	.7186	.7019	.7231	.7373
70	.5719	.5882	.6147	.6306
60	.4662	.4760	.4081	.4779
50	.3797	.4010	.3980	.4207

the runs for System 1, the 99 percent confidence interval contained the true reliability. It can be observed that, within the limits of the simulation error, as the sample size increases for all the systems, the confidence interval coverage improves. In those few cases where it did not improve, variability of the method can be seen.

There is a noticeable effect of the true system reliability on the confidence interval coverage. It appears that a lower system reliability corresponds to a more accurate confidence interval. System 1, with a reliability of .68448, has consistently more accurate confidence interval coverage than any other system. System 2 and 4, with reliabilities of .84297 and .84197 respectively, have table values that correspond closely. System 3, with the highest reliability, .99841, has the least accuracy.

As an indication of the time that was required to run these simulations on the CDC Cyber, Table VII specifies for all systems together, at different sample sizes, the average amount of time 12, 50 iteration program runs, to obtain 600 Monte Carlo iterations. That is, each program that was run contained all 4 systems at a given sample size, for 50 iterations.

Tables VIII through XI provide a comparison of the Modified Double Monte Carlo method with the: Univariate Method (Ref 15:33-34) for all systems at sample size 20;

TABLE VII

CPU Times on the CDC Cyber

Sample Size			
10	15	20	30
1456 sec	1871 sec	2378 sec	2982 sec

Bivariate Method (Ref 5:29) System 4 (only) sample size 20 for 700 Monte Carlo iterations.

For sample size 20, the modified Double Monte Carlo provides a more consistent coverage of the system reliability at confidence levels of 99, 95 and 90. At the remaining confidence levels, the univariate is more accurate. A reason for this is that since the distribution of the estimators for the parameters is unknown in the Double Monte Carlo, at the lower confidence limits the distribution is such that more reliability points appears in the distribution tails. It can also be noted that in most cases the confidence interval coverage is low for small sample sizes. Since there is high or optimistic system reliability estimates, the lower confidence limits are also too high and the confidence interval is not wide enough to cover the true system reliability.

Comparing the modified Double Monte Carlo method with the bivariate method, it can be seen that the bivariate

TABLE VIII

System 1 Comparison of Modified Double Monte Carlo and Univariate

Confidence Interval (Percent)	Univariate	Double Monte Carlo
99	.9617	.9910
95	.9050	.9371
90	.8617	.8702
80	.7800	.7689
70	.6817	.6709
60	.5700	.5011
50	.4833	.3788

TABLE IX

System 2 Comparison of Modified Double Monte Carlo and Univariate

Confidence Interval (Percent)	Univariate	Double Monte Carlo
99	.9383	.9709
95	.8600	.9213
90	.8083	.8671
80	.7200	.7118
70	.6467	.6177
60	.5433	.4890
50	.4700	.3900

TABLE X

System 3 Comparison of Modified Double Monte Carlo and Univariate

Confidence Interval (Percent)	Univariate	Double Monte Carlo
99	.9017	.9221
95	.7950	.7983
90	.7050	.6647
80	.5733	.4701
70	.4617	.3686
60	.3650	.2300
50	.2767	.1611

TABLE XI

System 4 Comparison of Modified Double Monte Carlo with Univariate and Bivariate

Confidence Interval (Percent)	Univariate	Double Monte Carlo	Bivariate
99	.9417	.9692	.996
90	.8033	.8600	.926
80	.7183	.7231	.826
70	.6450	.6147	.730
60	.5533	.4681	.604
50	.4733	.3980	.490

ethod is conservative, confidence interval is wider, thereby providing a more accurate confidence level that is much less sensitive to degradation due to high system reliability.

V. Conclusions and Recommendations

Conclusions

The modified Double Monte Carlo method developed in this thesis provides a useful tool for reliability estimation when there is a lack of component test data, that is, when there are limited test times to failure for a given component. The modified Double Monte Carlo demonstrates better results with respect to confidence levels for system reliability than the univariate case, when the sample size is small and a higher degree of confidence is required. As with the univariate case, if there is a high component or system reliability, the confidence intervals capture the true reliability to a lesser extent.

It can also be concluded that the skewness of the empirical distribution of the component reliabilities $\hat{\hat{R}}(t)$ has a definite impact on the results, to the extent that at the lower confidence intervals more reliabilities are outliers in the distribution tails.

The modified Double Monte Carlo method presented in this study will enable anyone to obtain approximate confidence intervals when the failure models are from the one, two or three parameter Weibull and the distribution of the estimators for the parameters are unknown.

Recommendations

The modified Double Monte Carlo should be used to investigate confidence levels when the component reliability, $R(t)$, failure times are modeled by the exponential, normal or lognormal distribution. Again, this should be attempted when the sample size is relatively small. Emphasis could also be placed on component reliabilities greater than 0.9.

Because of the limitation on the component empirical distribution, sensitivity analysis should be conducted to see what effect might take place when the distribution size is increased to 150, 200, and 300 points.

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Appendix A

Notes

The following notes pertain to the steps outlined in the modified Double Monte Carlo procedure in Chapter III.

Step 1: Weibull-distribution sample failure times were generated by using the International Mathematical and Statistical Libraries (IMSL) subroutine GGWIB.

Step 2: The Maximum Likelihood Estimators (MLE) of the shape and scale parameters ($\hat{k}$ and $\hat{\theta}$) were determined by use of a general solution subroutine of Weibull parameters called PARES (Ref 7). In this case, NSAM is the failure sample size, m = NSAM (i.e., there are no observations remaining after censoring), MR = 0 no observations are censored, R is the I-th order statistic of sample (I = 1, NSAM) . (Note: This iterative procedure, in the case of a Weibull population, is applicable to the most general case in which all three parameters are unknown and must be solved simultaneously. It is also applicable to special cases in which any one or any two of the parameters are unknown. This is accomplished by specifying combinations of ss1 , ss2 , or ss3 equal to one or zero.)

Steps 3, 4: Double Monte Carlo step where the component parameters $(\hat{\hat{k}},\hat{\hat{\theta}})$ are reestimated via MLE.

Step 5: The MLE of component reliability, $\hat{R}(t)$, was found by substitution of $\hat{\hat{k}}$ and $\hat{\hat{\theta}}$ into Eq (11).

Step 6: The bias of the MLE $\hat{R}(t)$ was determined by interpolation (using cubic splines) in Table I.

Step 7: The first and last order statistics of each component reliability were determined by linear extrapolation on the two nearest reliability points. Subroutine EXTRA is used for this purpose.

Step 8: A vector of 1000 component reliabilities is obtained from the empirical distribution using an IMSL subroutine GGUBS to generate uniform random deviates. Cubic splines are used to interpolate for the actual component reliabilities.

Step 10: The vectors of component reliabilities are combined using the system reliability equations [Eqs (13), (14), (15), and (16)]. This results in a vector of 1000 estimated system reliabilities for each system.

Step 11: These system reliability vectors are ordered in ascending sequence to obtain the 1, 5, 10, 20, 30, 40, 50 percentiles.

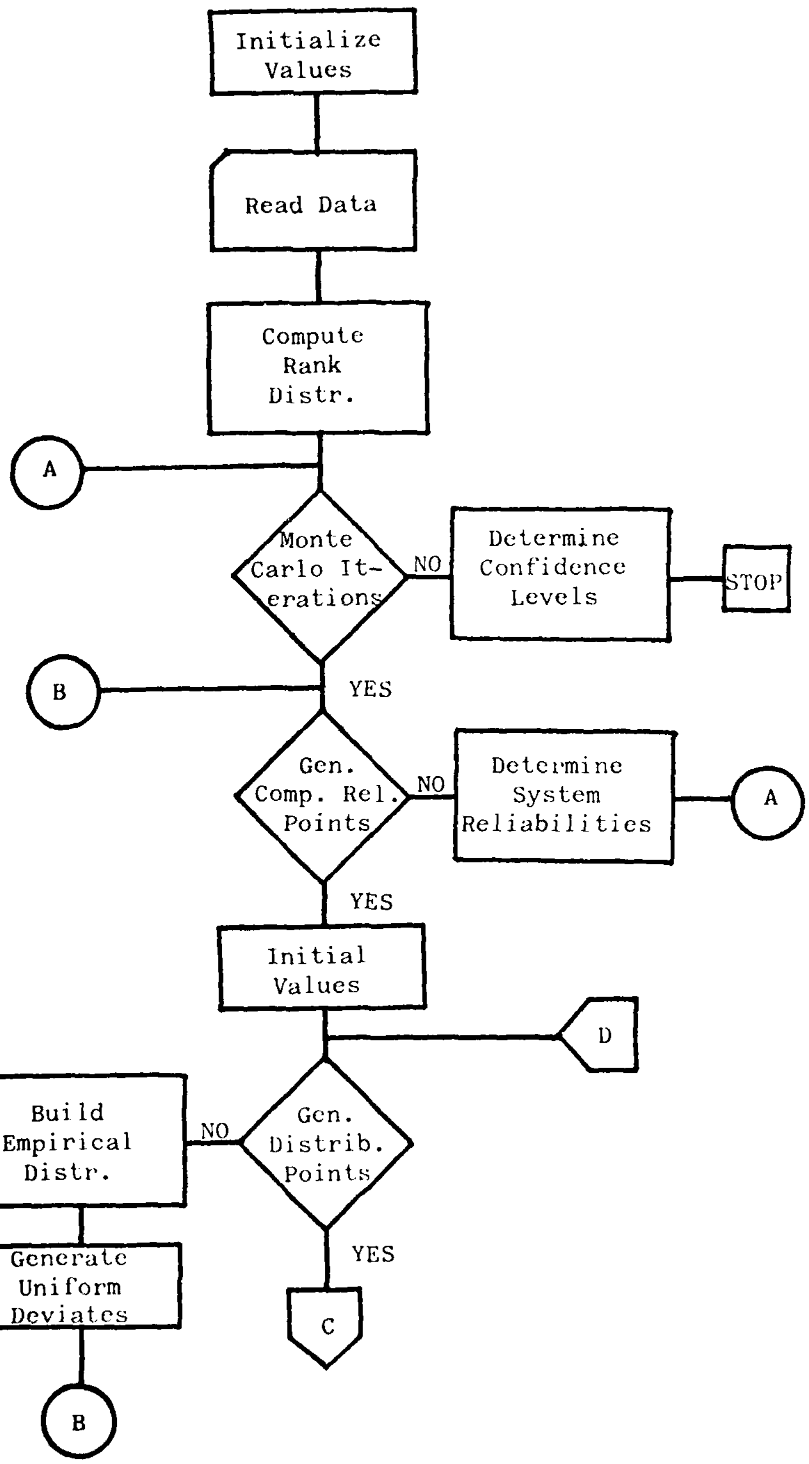
Step 13: If the lower confidence limit is less than or equal to the true system reliability then the interval contains the true reliability, subroutine CONLIM is used for this purpose.

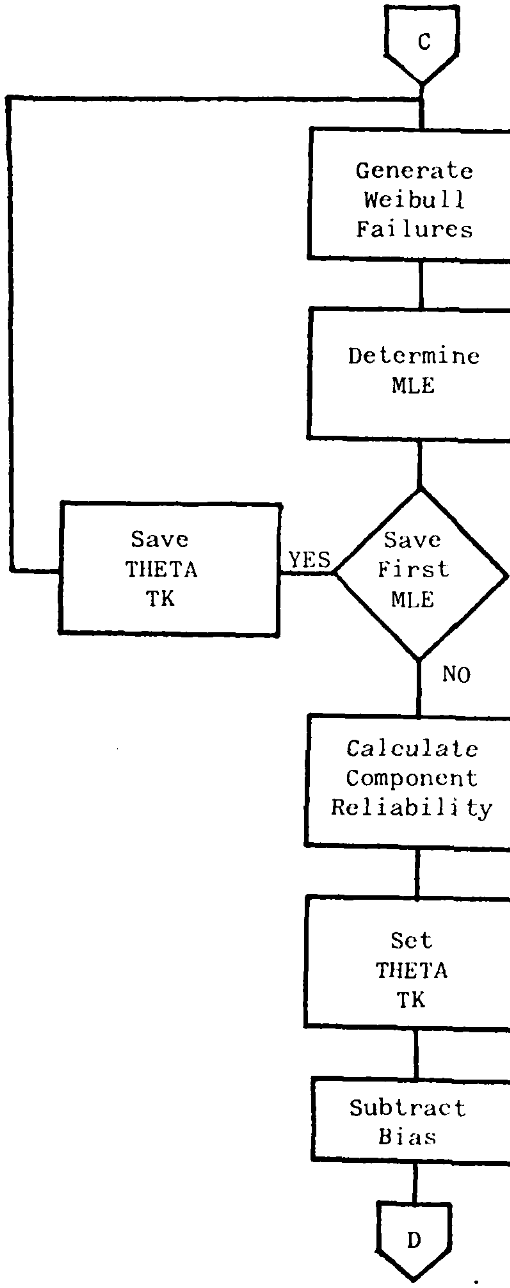
In this thesis, 500 Monte Carlo runs of the simulation were made in Step 12. For each run and system, it was noted if the true reliability was contained in the confidence

intervals. As a comparison, the X percent confidence interval should contain the true system reliability X percent of the time. This is the validation of the method.

Appendix B

Flow Diagram




C
Generate Weibull Failures
Determine MLE
Save First MLE
YES
Save THETA TK
NO
Calculate Component Reliability
Set THETA TK
Subtract Bias
D

Appendix C

Computer Program Listing

```
      PROGRAM DOUMC (INPUT, OUTPUT)
C
C  ******************************************************************
C  * THIS PROGRAM GENERATES, FOR EACH COMPONENT OF ANY SYSTEM, A     *
C  * SAMPLE SIZE NSAM OF FAILURE TIMES FROM THE WEIBULL DISTRIBUTION*
C  * USING THE TRUE PARAMETERS TK(SHAPE), TTHETA(SCALE) AND TC       *
C  * (LOCATION).  FROM THIS SAMPLE, THE MAXIMUM LIKELIHOOD           *
C  * ESTIMATORS (MLES) FOR K AND THETA ARE DERIVED USING HARTER AND *
C  * MOORE ITERATIVE SCHEME(PARES).  THESE ESTIMATES FOR K AND THETA*
C  * ARE THEN USED TO GENERATE A NEW SAMPLE SIZE NSAM OF WIEBULL     *
C  * FAILURES, WHICH ARE IN TURN USED TO RECALCULATE THE MLES FOR K *
C  * AND THETA.  THESE SECOND MLES ARE COMBINED TO YIELD CHAT, THE  *
C  * MLE FOR THE COMPONENT RELIABILITY.  BIASES FOR THESE ESTIMATES *
C  * ARE OBTAINED AND SUBTRACTED.  A GIVEN NUMBER (DSAM) OF THESE   *
C  * UNBIASED CHAT ARE ESTIMATED WHICH ARE USED TO FORM THE X-AXIS  *
C  * FOR AN IMIPRICAL DISTRIBUTION.  THE Y-AXIS IS OBTAINED USING A *
C  * MEDIAN RANK PROCEDURE.  THIS EMIPRICAL DISTRIBUTION IS THAN    *
C  * USED TO SAMPLE (NGEN) NUMBER OF COMPONENT RELIABILITIES.  THIS *
C  * PROCESS IS REPEATED FOR EACH COMPONENT AND THEN RELIABILITIES  *
C  * ARE COMBINED FOR 4 DIFFERENT SYSTEMS TO YIELD 4 VECTORS OF     *
C  * SAMPLE SYSTEM RELIABILITIES.  THESE VECTORS ARE THEN ORDERED   *
C  * AND THE 99,95,90,80,70,60,50 PERCENT LOWER CONFIDENCE LIMITS   *
C  * ARE PICKED (CONLIM).  IT IS NOTED IF THE TRUE RELIABILITY IS   *
C  * CONTAINED WITHIN THESE INTERVALS.                              *
C  * THE ABOVE PROCESS IS REPEATED FOR NOLMC MONTE CARLO RUNS WITH  *
C  * COUNTERS FOR EACH SYSTEM TO TRACK THE NUMBER OF TIMES THAT THE *
C  * TRUE RELIABILITY IS CAPTURED.                                  *
C  ******************************************************************
      DIMENTION ARCHAT(1000,5),ATRS(4),BI(12,15),BPAR(4),C(4,4),
     1CCHAT(100),PARAM(3,5),R(500,5),RANK(100),RLBS(12),SCHAT(1000),
     2SMSZ(15),SPLINE(99,3),TEMP(100),TRC(5),TRS(1),UNIF(1000),
     3WK(200),X(2),ZCHAT(100),CC(550),THETA(550),EK(550)
       DOUBLE PRECISIONDSEED
      INTEGER DSAM
      DATA BPAR/0.,0.,0.,0./
       DATA RLBS/.5,.55,.6,.65,.7,.75,.8,.85,.9,.925,.95,
     1   .98/
       DATA SMSZ/8.,9.,10.,11.,12.,13.,14.,15.,20.,25.,
     1   30.,40.,50.,75.,100./
C
C  INITIALIZE VARIABLES***
      READ*,DSEED,T
      CALL RANSET(T)
      TIME=100.
      NRLBS=12
      NSMSZ=15
      NMC1=2
      NSMSZ1=NSMSZ-1
      NRLBS1=NRLBS-1
      ISD=12
      NGEN=1000
```

```
      NS1C99 = 0
      NS1C95 = 0
      NS1C90 = 0
      NS1C80 = 0
      NS1C70 = 0
      NS1C60 = 0
      NS1C50 = 0
      NS2C99 = 0
      NS2C95 = 0
      NS2C90 = 0
      NS2C80 = 0
      NS2C70 = 0
      NS2C60 = 0
      NS2C50 = 0
      NS3C99 = 0
      NS3C95 = 0
      NS3C90 = 0
      NS3C80 = 0
      NS3C70 = 0
      NS3C60 = 0
      NS3C50 = 0
      NS4C99 = 0
      NS4C95 = 0
      NS4C90 = 0
      NS4C80 = 0
      NS4C70 = 0
      NS4C60 = 0
      NS4C50 = 0
C
      WRITE 400
C
C  READ SAMPLE SIZE AND NUMBER OF MONTE CARLO RUNS***
C
      READ *,NSAM,NOLMC,DSAM
      WRITE 401,NSAM,NOLMC,DSAM
C
C  SET PARAMETERS FOR PARES AND RANK DISTRIBUTION***
C
      M=NSAM
      MR=0
      RNSAM=NSAM
      NNMC2=DSAM-2
      TSAM=DSAM
      IC=DSAM-1
C
C  READ TRUE COMPONENT PARAMETERS***
C
      READ *,((PARAM(I,J),I=1,3),J=1,5)
      WRITE 402,((J,(PARAM(I,J),I=1,3),CMPREL(TIME,PARAM(1,J),
     1PARAM(2,J),PARAM(3,J))),J+1,5)
C
C  READ BIAS LEVELS***
C
```

```
      READ *,((BI(I,J),J=1,15),I=1,12)
      WRITE 500
      WRITE 403,((BI(I,J),J=1,15),I=1,12)
C
C  CALCULATE MEDIAN RANK DISTRIBUTION***
C
      RANK(1)=0.
      RANK(DSAM)=1.
      DO 305 I=1,NNMC2
      RANK(I+1)=(I-.3)/(TSAM+.4)
 305  CONTINUE
C*********************************************************************
C*********************************************************************
C  THIS LOOP (FROM HERE TO STATEMENT 300) COMPLETES NOLMC MONTE CARLO
C  RUNS OF THE PROGRAM***
C
      DO 300 NCOUNT=1,NOLMC
C
C*********************************************************************
C  FOR EACH OF 5 COMPONENTS, THIS LOOP GENERATES THE MLE
C  OF CHAT FROM THE MLES OF K AND THETA.  IT SUBTRACTS THE
C  BIAS: ESTABLISHES THE EMIPRICAL DISTRIBUTION AND SAMPLES
C  THE GIVEN NUMBER OF COMPONENT RELIABILITIES***
      DO 200 J=1,5
      TK=PARAM(1,J)
      TC=PARAM(3,J)
C
C  DETERMINE TRUE COMPONENT RELIABILITY***
      TRC(J)=CMPREL(TIME,TK,TTHETA,TC)
C
C  THIS LOOP GENERATES THE DESIRED NUMBER OF EMIPRICAL
C  DISTRIBUTION POINTS***
C
      DO 150 LL=1,NNMC2
      LK=LL
C
C  GATHER NSAM FAILURE TIMES FROM THE WEIBULL
C  DISTRIBUTION WITH TRUE PARAMETERS TK,TTHETA
C  AND TC***
 220  CALL GGWIB(DSEED,TK,NSAM,TEMP)
      DO 201 I=1,NSAM
      R(I,J)=TTHETA*TEMP(I)+TC
 201  CONTINUE
      CC(1)=TC
      THETA(1)=TTHETA
      EK(1)=TK
C
C  DETERMINE THE MLES FOR THETA AND K***
      CALL PARES(NSAM,M,CC,THETA,EK,MR,TTHETA,TK,R(1,J))
      IF(LK.NE.1)GO TO 215
      X(1)=TTHETA
      X(2)=TK
      LK=LK+1
      GO TO 220
```

```
C
C  DETERMINE THE COMPONENT RELIABILITY USING THE TRUE C
C  AND THE TWICE MLES OF K AND THETA***
 215  CHAT=CMPREL(TIME,TK,TTHETA,TC)
      TTHETA=X(1)
      TK=X(2)
C
C  GIVEN THE MLE OF THE COMPONENT RELIABILITY AND THE
C  SAMPLE SIZE, ENTER THE 2-DIMENSIONAL ARRAY BI
C  AND FIND THE BIAS OF THE ESTIMATOR***
      DO 203 I=1,NSMSZ1
      LSMSZ=1
      IF(RNSAM.LE.SMSZ(I+1))GO TO 204
 203  CONTINUE
      GO TO 205
 204  IF(CHAT.LT..5)GO TO 206
      DO 207 I=1,NRLBS1
      LRLBS+I
      IF(CHAT.LE.RLBS(I+1))GO TO 208
 207  CONTINUE
      GO TO 205
 208  CALL IBCICU(BI,ISD,RLBS,NRLBS,SMSZ,NSMSZ,LRLBS,LSMSZ,C,WK,IER)
      CALL IBCEVU(RLBS,NRLBS,SMSZ,NSMSZ,LRLBS,LSMSZ,C,CHAT,RNSAM,
     1BIAS,IER)
      IF(LRLBS.LT.NRLBS)GO TO 209
 205  BIAS=0.
      GO TO 209
 206  IF(RNSAM.EQ.SMSZ(LSMSZ+1))LSMSZ=LSMSZ+1
      BIAS=BI(1,LSMSZ)
C
C  SUBTRACT THE BIAS***
 209  ZCHAT(LL)=CHAT-BIAS
 150  CONTINUE
C
C  ORDER THE RELIABILITIES FOR THE EMIPRICAL DISTRIBUTION***
      CALL VSRTA(ZCHAT,NNMC2)
      DO 211 I=1,NNMC2
 211  CCHAT(I+1)=ZCHAT(I)
C
C  FIND THE FIRST AND LAST ORDER STATISTICS FOR THE
C  EMIPRICAL DISTRIBUTION COMPONENT RELIABILITIES***
      CALL EXTRA(CCHAT,RANK,NMC1,0.,T2,DSAM)
      CCHAT(1)=T2
      IF(T2.LE.0.)CCHAT(1)=0.
      CALL EXTRA(CCHAT,RANK,NNMC2,1.,T2,DSAM)
      CCHAT(DSAM)=T2
      IF(T2.GE.1.)CCHAT(DSAM)=1.
C
C  SAMPLE FROM A UNIFORM GENERATOR (NGEN) NUMBER OF
C  COMPONENT RELIABILITIES***
      CALL GGUBS(DSEED,NGEN,(NIF)
      CALL ICSICU(RANK,CCHAT,DSAM,BPAR,SPLINE,IC,IER)
      CALL ICSEVU(RANK,CCHAT,DSAM,SPLINE,IC,UNIF,SCHAT,NGEN,IER)
      DO 210 KK=1,NGEN
```

```
 210  ARCHAT(KK,J)=SCHAT(KK)
 200  CONTINUE
C
C  FOR EACH OF THE 4 SYSTEMS, THE DIFFERENT COMPONENTS ARE
C  COMBINED TO YIELD(NGEN) SAMPLES OF THE SYSTEM RELIABILITIES.
C  THESE SAMPLES ARE SEQUENCED IN ASCENDING ORDER THE
C  COUNTERS KEEP TRACK OF WHEN THE 99,95,90,80,70,60,50
C  PERCENT CONFIDENCE INTERVALS CONTAIN THE TRUE SYSTEM
C  RELIABILITIES***
C
C  SYSTEM 1***
      NMC=1
      CALL REL1(NMC,TRC,TRS)
      ATRS(1)=TRS(1)
      CALL REL1(NGEN,ARCHAT,SCHAT)
      CALL VSRTA(SCHAT,NGEN)
      CALL CONLIM(SCHAT,TRS,NS1C99,NS1C95,NS1C90,NS1C80,NS1C70,NS1C60,
     1NS1C50,NGEN)
C
C  SYSTEM 2***
      NMC=1
      CALL REL2(NMC,TRC,TRS)
      ATRS(2)=TRS(1)
      CALL REL2(NGEN,ARCHAT,SCHAT)
      CALL VSRTA(SCHAT,NGEN)
      CALL CONLIM(SCHAT,TRS,NS2C99,NS2C95,NS2C90,NS2C80,NS2C70,NS2C60,
     1NS2C50,NGEN)
C
C  SYSTEM 3***
      NMC=1
      CALL REL3(NMC,TRC,TRS)
      ATRS(3)=TRS(1)
      CALL REL3(NGEN,ARCHAT,SCHAT)
      CALL VSRTA(SCHAT,NGEN)
      CALL CONLIM(SCHAT,TRS,NS3C99,NS3C95,NS3C90,NS3C80,NS3C70,NS3C60,
     1NS3C50,NGEN)
C
C  SYSTEM 4***
      NMC=1
      CALL REL4(NMC,TRC,TRS)
      ATRC(4)=TRS(1)
      CALL REL4(NGEN,ARCHAT,SCHAT)
      CALL VSRTA(SCHAT,NGEN)
      CALL CONLIM(SCHAT,TRS,NS4C99,NS4C95,NS4C90,NS4C80,NS4C70,NS4C60,
     1NS4C50,NGEN)
 300  CONTINUE
C
        RNOLMC = NOLMC
C  FOR SYSTEM 1, DETERMINE THE 95, 90, AND 80 PERCENT CONFIDENCE
C  LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
        PS1C99 = NS1C99/RNOLMC
        PS1C95 = NS1C95/RNOLMC
        PS1C90 = NS1C90/RNOLMC
        PS1C80 = NS1C80/RNOLMC
```

```
      PS1C70 = NS1C70/RNOLMC
      PS1C60 = NS1C60/RNOLMC
      PS1C50 = NS1C50/RNOLMC
C  FOR SYSTEM 2, DETERMINE THE 95, 90, AND 80 PERCENT CONFIDENCE
C  LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS2C99 = NS2C99/RNOLMC
      PS2C95 = NS2C95/RNOLMC
      PS2C90 = NS2C90/RNOLMC
      PS2C80 = NS2C80/RNOLMC
      PS2C70 = NS2C70/RNOLMC
      PS2C60 = NS2C60/RNOLMC
      PS2C50 = NS2C50/RNOLMC
C  FOR SYSTEM 3, DETERMINE THE 95, 90, AND 80 PERCENT CONFIDENCE
C  LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS3C99 = NS3C99/RNOLMC
      PS3C95 = NS3C95/RNOLMC
      PS3C90 = NS3C90/RNOLMC
      PS3C80 = NS3C80/RNOLMC
      PS3C70 = NS3C70/RNOLMC
      PS3C60 = NS3C60/RNOLMC
      PS3C50 = NS3C50/RNOLMC
C  FOR SYSTEM 4, DETERMINE THE 95, 90, AND 80 PERCENT CONFIDENCE
C  LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS4C99 = NS4C99/RNOLMC
      PS4C95 = NS4C95/RNOLMC
      PS4C90 = NS4C90/RNOLMC
      PS4C80 = NS4C80/RNOLMC
      PS4C70 = NS4C70/RNOLMC
      PS4C60 = NS4C60/RNOLMC
      PS4C50 = NS4C50/RNOLMC
     PRINT 404
     PRINT 405,ATRS(1),PS1C99,PS1C95,PS1C90,PS1C80,PS1C70,PS1C60,PS1C50
     PRINT 406
     PRINT 405,ATRS(2),PS2C99,PS2C95,PS2C90,PS2C80,PS2C70,PS2C60,PS2C50
     PRINT 407
     PRINT 405,ATRS(3),PS3C99,PS3C95,PS3C90,PS3C80,PS2C70,PS2C60,PS2C50
     PRINT 408
     PRINT 405,ATRS(4),PS4C99,PS4C95,PS4C90,PS4C80,PS4C70,PS4C60,PS4C50
     STOP
C
C*********************************************************
C*********************************************************
C400  FORMAT("1")
 401   FORMAT ( " *******************************************"
    1   "**********", / , " *", T62, "*", / , " *", T62, "*", /
    2   , " *", T25, "SAMPLE " "SIZE = ", I3, T62, "*", / . " *",
    3   T62, "*", / , " *", T22, "MONTE CARLO " "SIZE   ", I3,/,
    4    T62,"*",/, "*", T21, "DISTRIBUTION SIZE =  ", I3,
    5   T62, "*", / , " *", T62, "*", / , " *", T62, "*", / ,
    6   " *************************************************"
    7   "*", / / / / / / / / )
 402   FORMAT ( 5(1X, "COMPONENT ", I1, / , 6X, "K   ", F4.2, / ,
    1   6X, "THETA = ", F5.0, / , 6X, "C = ", F2.0, / , 6X,
    2   "RELIABILITY = ", F7.5, / / ) )
```

```
403   FORMAT ( 15(1X, F7.5) )
404   FORMAT ( / " ***** SYSTEM 1 *****" / " (3 COMPONENTS IN SERI"
    1   "ES)" )
405   FORMAT ( / , " TRUE SYSTEM RELIABILITY =", F7.5, /
    1   " THE 99 PERCENT CONFIDENCE INTERVAL COVERED ", F6.4,
    2   " OF THE RUNS", / , " THE 95 PERCENT CONFIDENCE INTERVAL COV"
    3   "ERED ", F6.4 "OFTHERUNS", / , " THE 90 PERCENT CONFIDENCE I"
    4   "NTERVAL COVERED " F6.4, " OF THE RUNS", / , " THE 80 PERCE"
    5   "NT  CONFIDENCE  INTERVAL" "COVERED " F6.4 "OFTHERUNDS", / ,
    6   " THE 70 PERCENT CONFIDENCE " "INTERVAL COVERED ", F6.4,
    7   " OF THE RUNS", / , " THE 60 PERCENT " "CONFIDENCE INTERVAL"
    8   " COVERED ", F6.4, " OF THE RUNDS", / , " THE", " 50 PERCEN"
    9   "T CONFIDENCE INTERVAL COVERED ", F6.4, " OF THE RUNS", / /
    9   / )
406   FORMAT ( / " ***** SYSTEM 2 *****" / " (1 COMPONENT IN SERIE"
    1   "S WITH 2 " "IN PARALLEL)" )
407   FORMAT ( / " ***** SYSTEM 3 *****" / " (3 COMPONENTS IN PARA"
    1   "LLEL)" )
408   FORMAT ( / " ***** SYSTEM 4 *****" / " (A 5-COMPONENT COMPLE"
    1   "X NETWORK)" )
500  FORMAT (" ", "BIAS OF ESTIMATED COMPONENT RELIABILITY")
     END
```

```
      FUNCTION CMPREL(TIME,K,THETA,C)
C
C  THIS FUNCTION ROUTINE CALCULATES THE COMPONENT
C  RELIABILITIES FROM THE WEIBULL DISTRIBUTION***
      REAL K
      CMPREL=EXP(-((TIME-C)/THETA)**K)
      RETURN
      END
```

```
      SUBROUTINE REL1(NMC,R,RS)
C  REL1 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN SERIES***
      DIMENSION R(NMC,5), RS9NMC)
      DO 10 I=1,NMC
   10 RS(I)=R(I,1) * R(I,2) * R(I,3)
      RETURN
      END
```

```
      SUBROUTINE REL2(NMC,R,RS)
C  REL2 DETERMINES THE SYSTEM RELIABILITY OF 1 COMPONENT IN SERIES
C  WITH 2 IN PARALLEL***
      DIMENSION R(NMC,5), RS(NMC)
      DO 10 I=1,NMC
   10 RS(I)=R(I,1)*(1.-(1.-R(I,2))*(1,-R(I,3)))
      RETURN
      END
```

```
      SUBROUTINE REL3(NMC,R,RS)
C  REL3 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN PARALLEL***
      DIMENSION R(NMC,5) , RS(NMC)
      DO 10 I=1,NMC
   10 RS(I)=1.-(1.-R(I,1))*(1.-R(I,2))*(1.-R(I,3))
      RETURN
      END
```

```
      SUBROUTINE REL4(NMC,R,RS)
C  REL4 DETERMINES THE SYSTEM RELIABILITY OF A 5 COMPONENT COMPLEX
C  NETWORK***
      DIMENSION R(NMC,5), RS(NMC)
      DO 10 I=1,NMC
      RS(I)=R(I,1)*(1.-(1.-R(I,2))*(1.-R(I,5)*(1.-(1.-R(I,3))
     1*(1.-R(I,4)))))
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EXTRA(X,Y,I,T1,T2,NNMC)
C
C  THIS SUBROUTINE IS USED TO APPROXIMATE THE FIRST AND LAST ORDER
C  STATISTICS OF THE COMPONENT RELIABILITY FOR THE EMIPRICAL
C  DISTRIBUTION***
      DIMENSION X(NNMC),Y(NNMC)
      SLOPE=(Y(I+1)-Y(I))/(X(I+1)-X(I))
      B=Y(I)-SLOPE*X(I)
      T2=(T1-B)/SLOPE
      RETURN
      END
```

```
      SUBROUTINE CONLIM(R,RS,N1,N2,N3,N4,N5,N6,N,7,NMC)
C
C  THIS SUBROUTINE KEEPS TRACK OF THE NUMBER OF TIMES THE TRUE
C  RELIABILITIES IS CONTAINED WITHIN A GIVEN CONFIDENCE INTERVAL***
      DIMENSION R(NMC),RS(1)
      IF(R(10) .LE. RS(1)) N1=N1+1
      IF(R(50) .LE. RS(1)) N2=N2+1
      IF(R(100) .LE. RS(1)) N3=N3+1
      IF(R(200) .LE. RS(1)) N4=N4+1
      IF(R(300) .LE. RS(1)) N5=N5+1
      IF(R(400) .LE. RS(1)) N6=N6+1
      IF(R(500) .LE. RS(1)) N7=N7+1
      RETURN
      END
```

```
      SUBROUTINE PARES(N,M,C,THETA,EK,MR,PTH,PEK,T)
C     INPUT
C     N=SAMPLE SIZE (BEFORE CENSORING),N=100 OR LESS AS
C             DIMENSIONED
C     SS1=0 IF SCALE PARAMETER THETA IS KNOWN
C     SS1=1 IF SCALE PARAMETER THETA IS TO BE ESTIMATED
C     SS2=0 IF SHAPE PARAMETER K IS KNOWN
C     SS2=1 IF SHAPE PARAMETER K IS TO BE ESTIMATED
C     SS3=0 IF LOCATION PARAMETER C IS KNOWN
C     SS3=1 IF LOCATION PARAMETER C IS TO BE ESTIMATED
C     T(I)=I-TH ORDER STATISTIC OF SAMPLE (I=1,N)
C     M=NUMBER OF OBSERVATIONS REMAINING AFTER CENSORING N-M
C                 FROM ABOVE
C     C(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF C
C     THETA(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF THETA
C     EK(1)=INITIAL ESTIMATE (OR KNOWN VALUE) OF K
C     MR=NUMBER OF OBSERVATIONS CENSORED FROM BELOW
C     OUTPUT
C     N,SS1,SS2,SS3,M,C(1),THETA(1),EK(1),MR
C             --SAME AS FOR INPUT
C     C(J)=ESTIMATE AFTER J-1 ITERATIONS
C              (OR KNOWN VALUE) OF C
C     THETA(J)=ESTIMATE AFTER J-1 ITERATIONS
C              (OR KNOWN VALUE) OF THETA
C     EK(J)=ESTIMATE AFTER J-1 ITERATIONS
C           (OR KNOWN VALUE) OF K
C       (MAXIMUM VALUE OF J AS PRESENTLY DIMENSIONED IS 500)
C     EL=NATURAL LOG. OF LIKELIHOOD FOR C(J),THETA(J),EK(J)
      DIMENSION T(500),C(500),THETA(550),EK(550),X(56),Y(55)
      SS1=1.
      SS2=1.
      SS3=0.
      IF(N)66,66,104
  104 EN=N
      IF(M)66,66,110
  110  EM=M
  31  ELNM=0.
      EMR=MR
      MRP=MR+1
  33  NM=N-M+1
      DO 34 I=NM,N
      EI=I
  34  ELNM=ELNM+ALOG(EI)
      IF (MR) 66,35,74
  74  DO 75 I=1,MR
      EI=I
  75  ELNM=ELNM-ALOG(EI)
  35  DO 30 J=1,550
      IF (J-1) 66,25,37
  37  JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
  6   SK=SK+(T(I)-C(JJ))**EK(JJ)
```

```
      IF(SS1) 7,7,8
7     THETA(J)=THETA(JJ)
      GO TO 9
8     IF (MR) 66,19,20
19    THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ)**EK(JJ)/EM)
     C**(1./EK(JJ))
      GO TO 9
20    X(1)=THETA(JJ)
      LS=0
      DO 21 L=1,55
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
      Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)
     C+EK(JJ)*(EN-EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.
     C)-EMR*EK(JJ)*ZRK*EXP(-ZRK)/(X(L)*(1.-EXP(-ZRK)))
      IF (Y(L)) 53,73,54
53    LS=LS-1
      IF (LS+L) 58,55,58
54    LS=LS+1
      IF (LS-L) 58,56,58
55    X(LP)=.5*X(L)
      GO TO 61
56    X(LP)=1.5*X(L)
      GO TO 61
58    IF (Y(L)*Y(LL)) 60,73,59
59    LL=LL-1
      GO TO 58
60    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
61    IF (ABS(X(LP)-X(L))-1.E-4) 73,73,21
21    CONTINUE
73    THETA(J)=X(LP)
9     EK(J)=EK(JJ)
10    IF (SS2) 12,12,11
11    DO 17 I=MRP,M
17    SL=SL+ALOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,55
      SLK=0.
      DO 18 I=MRP,M
18    SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))
     C**X(L)
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)
     C**X(L)+(EN-EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)
     C-C(JJ))**X(L)/THETA(J)**X(L)+EMR*ZRK*(ALOG(ZRK)/X(L))
     C*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF(Y(L)) 43,52,44
43    LS=LS-1
```

```
      IF(LS+L) 47,45,47
   44 LS=LS+1
      IF (LS-L) 47,46,47
   45 X(LP)=.5*X(L)
      GO TO 50
   46 X(LP)=1.5*X(L)
      GO TO 50
   47 IF (Y(L)*Y(LL)) 49,52,48
   48 LL=LL-1
      GO TO 47
   49 X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
   50 IF (ABS(X(LP)-X(L))-1.E-4) 52,52,51
   51 EK(J)=X(LP)
   12 C(J)=C(JJ)
   62 IF (SS3) 25,25,14
   14 IF (1.-EK(J)) 16,78,78
   78 IF (SS1+SS2) 57,57,16
   16 X(1)=C(J)
      LS=0
      DO 23 L=1,55
      SK1=0.
      SR=0.
      DO 15 I=MRP,M
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
   15 SR=SR+1./(T(I)-X(LL))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))
     C**(EK(J)-1.))/THETA(J)**EK(J)-EMR*EK(J)*ZRK(EXP(-ZRK)
     C/((T(MRP)-X(L))*(1.-EXP(-ZRK)))
      IF (Y(L)) 39,24,40
   39 LS=LS-1
      IF (LS+L) 70,41,70
   40 LS=LS+1
      IF (LS-L) 70,42,70
   41 X(LP)=.5*X(L)
      GO TO 22
   42 X(LP)=.5*X(L)+.5*T(1)
      GO TO 22
   70 IF (Y(L)*Y(LL)) 72,24,71
   71 LL=LL-1
      GO TO 70
   72 X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
   22 IF (ABS(X(LP)-X(L))-1.E-4) 24,24,23
   23 CONTINUE
   24 C(J)=X(LP)
      GO TO 25
   57 C(J)=T(1)
   25 IF (MR) 66,38,69
   38 DO 63 I=1,M
      IF (C(J)+1.E-4-T(I)) 68,67,67
   67 MR=MR+1
```

```
63  C(1)=T(1)
68  IF(MR) 66,69,31
69  SK=0.
    SL=0.
    DO 36 I=MRP,M
    SK=SK+(T(I)-C(J))**EK(J)
36  SL=SL+ALOG(T(I)-C(J))
    ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
    EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+
   C(EK(J)-1.)*SL-(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA
   C(J)**EK(J))+EMR*ALOG(1.-EXP(-ZRK))
150  IF(J-3) 30,27,27
27  IF (ABS(C(J)-C(JJ))-1.E-4) 28,28,30
28  IF (ABS(THETA(J)-THETA(JJ))-1.E-4) 29,29,30
29  IF(ABS(EK(J)-EK(JJ))-1.E-4)126,126,30
30  CONTINUE
126 PTH=THETA(J)
    PEK=EK(J)
    GO TO 140
66  PRINT 135
135 FORMAT(1H ,20HAL SAMPLES CENSORED,/)
    PEK=0.
    PTH=0.
140 CONTINUE
    RETURN
    END
```

VITA

James Ward Johnston, Jr., was born on 28 September 1950 in San Diego, California. He graduated from Point Loma High School in San Diego in 1968 and attended San Diego State University, where upon graduation in 1973 he received a bachelor's degree in Mathematics. He then worked as a cadastrial draftsman for the County of San Diego for a period of one year. He then completed training at Officers Training School at Lackland AFB TX, and was commissioned a second lieutenant in the United States Air Force. His initial assignment was to the E-4 System Program Office at Hanscom AFB MA, where he served as a Computer Design Engineer. After approximately four years at Hanscom AFB, he entered the School of Engineering, Air Force Institute of Technology, in June 1979.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A modified Double Monte Carlo technique is developed for determination of lower confidence limits of system reliability based on component test data. It is assumed that the components test data consists of failure times, which are distributed according to a known two-parameter Weibull probability distribution. These failure times are randomly generated using the true shape and scale parameters of the distribution. Maximum-likelihood

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estimators are twice obtained for the shape and scale parameters and then substituted into the reliability equation to obtain an -stimator for the component reliability. A given number of these estimators are obtained and used to form an empirical distribution of reliabilities for each component. A given number of samples from this distribution are used to calculate various system reliabilities. Since the true system reliability is known, it can be determined if a given confidence interval contain the true number, hence giving you a method of validating any confidence interval.

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